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A NOVEL REINITIALIZATION TECHNIQUE TO CONSERVE MASS AND ENHANCE ACCURACY IN VOF METHOD

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ABSTRACT

A novel reinitialization technique for mass conservation and reduction of diffusion error in VOF method is presented. This technique is general enough to be easily extended to unstructured mesh. The efficacy of the present technique is demonstrated by computing a few standard test problems.

Key Words: *Volume of Fluid, Reinitialization, Mass Conservation, Sharp Interface Capturing.*

1. INTRODUCTION

The volume of fluid (VOF) method is one of the very popular approaches for solving multi-fluid flows. In this method, accurate positioning of fluid interface plays a vital role as it allows accurate calculation of surface forces and realistic visualization of the flow. In this method, an advection equation of volume fraction C, given by equation,

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = 0 \tag{1}$$

is added to the Navier-Stokes equations to solve a two-fluid system. This equation is solved in volume tracking methods, like VOF, for interface capturing. The piecewise linear interface calculation (PLIC) technique in VOF has been very successful for interface capturing which is extensively studied in [1, 4]. The PLIC technique is not simple to extend to unstructured meshes due to its dependence on mesh geometry for flux calculation [2]. Therefore, the need for a more elegant and general approach of reconstruction of fluid interface and calculation of interface fluxes is felt. An alternative approach may be to use Riemann solver with solution reconstruction. However, the major drawback with this approach is the high diffusion at fluid interface with lower order solution reconstruction. Also, high-order non-total-variation-diminishing (non-TVD) method can introduce spurious oscillations in the solution near discontinuities (fluid interfaces). In addition, the very-high-order methods are computationally expensive. A new method for maintaining the sharp fluid interface is therefore presented here, which can be used with moderate order of solution reconstruction approach along with a Riemann solver. The new reinitialization technique is tested with a first order and well-known high-order solution reconstructions based on Weighted Essentially Non-Oscillatory (WENO) approach of third and fifth order of accuracy [5].

2. NOVEL REINITIALIZATION TECHNIQUE

The reinitialization of VOF function is based on the marching squares method [3]. The marching squares method is widely used in computer graphics visualization for producing contour lines and it can be easily extended to any cell geometry. This method with few modifications is adopted here for calculation of total mass for a given volume fraction level. In this approach, we begin by setting total mass equal to zero and interpolating the cell center values to the cell nodes. Every cell node is then evaluated to check whether its value is above or below the volume fraction level and it is marked as 1 or 0 respectively. The cell which has all four nodes marked as 1 is contained inside the volume fraction, therefore the area of the cell is added to total mass. On the other hand, the cell with all nodes marked as 0 are ignored. All the remaining cells contain the interface line corresponding to the volume fraction being considered. These cells fall in one of the 14 cases (case 1 through 14) as shown in FIGURE 1 and the appropriate mass (area in 2D) is added to total mass. The calculated total mass obviously may not be equal to the exact mass of the tracked fluid. Therefore, we start with 0.5 as the initial guess value, and iterate to converge to the volume fraction level corresponding to the exact mass using the bisection method up to an accuracy of 10^{-6}. Once the volume fraction level which conserves the mass is obtained, the volume fraction field is reinitialized with the volume fraction obtained from FIGURE 1. It may be noted that, frequent reinitialization of VOF field is not required, especially in case of high order methods, as the diffusion is well controlled, and unnecessary reinitialization adds to the computational overhead. Also, less frequent reinitialization will lead to accumulation of diffusion error. Hence, there exists an optimal reinitialization frequency for a given problem depending on the method of solution reconstruction.

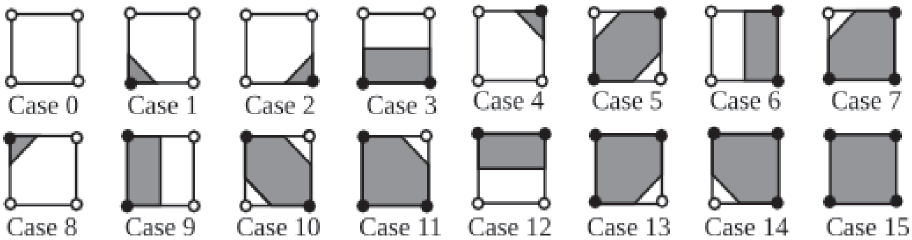


FIGURE 1. Marching Squares Algorithm

3. ERROR CALCULATION

Let the exact solution of equation (1) for the i^{th} cell be C_i^e and the numerical solution be C_i^n. The L_1- and L_2-norm of error in the solution for the i^{th} cell can then be defined as,

$$\epsilon_{L1} = \frac{1}{N}\sum^{N} \epsilon_i ; \quad \epsilon_{L2} = \sqrt{\frac{1}{N}\sum^{N} (\epsilon_i)^2} ; \quad \text{where,} \quad \epsilon_i = |C_i^e - C_i^n|$$

4. RESULTS AND DISCUSSION

The performance of the new reinitialization technique is demonstrated by solving two standard test problems. The contour plots of reinitialized solution are plotted only for a reinitialization frequency of 40 iterations and for a mesh size of 160×160.

Test Problem 1 - Translation of Square Volume: A square volume of side 0.2 is placed in a uniform velocity field given by $\vec{V} = (1, -1)$. The computational domain has dimensions of $[0,1] \times [0,1]$, and the square volume is placed at time $t = 0$ with its left-bottom corner at $(0.4, 0.4)$. The contour plots of the VOF function for different reconstruction methods are shown in FIGURE 2 with the initial position of the square shown using dashed line. The error plots and the computing time plots are shown in FIGURE 3 and FIGURE 4 respectively.

Test Problem 2 - Circular Volume in Transient Shear Velocity Field: A circular volume of radius 0.2π is placed in a transient shearing velocity field given by,

$$\vec{V}(x, y, t) = \begin{cases} (\sin(x)\cos(y), -\cos(x)\sin(y)), & t < T/2 \\ (-\sin(x)\cos(y), \cos(x)\sin(y)), & t \geq T/2 \end{cases} ; \ \text{where}, T = 8$$

The computational domain has dimensions of $[0, \pi] \times [0, \pi]$, and the circular volume is placed at time $t = 0$ with its center at $(0.5\pi,\ 0.2\ +\ 0.2\pi)$. The contour plots of final solution (at $t = T = 8$) of the VOF function for different reconstruction methods and the maximum extent of shear (shown as dashed line at $t = 4$) are shown in FIGURE 5. The error plots and the computing time plots are shown in FIGURE 6 and FIGURE 7 respectively.

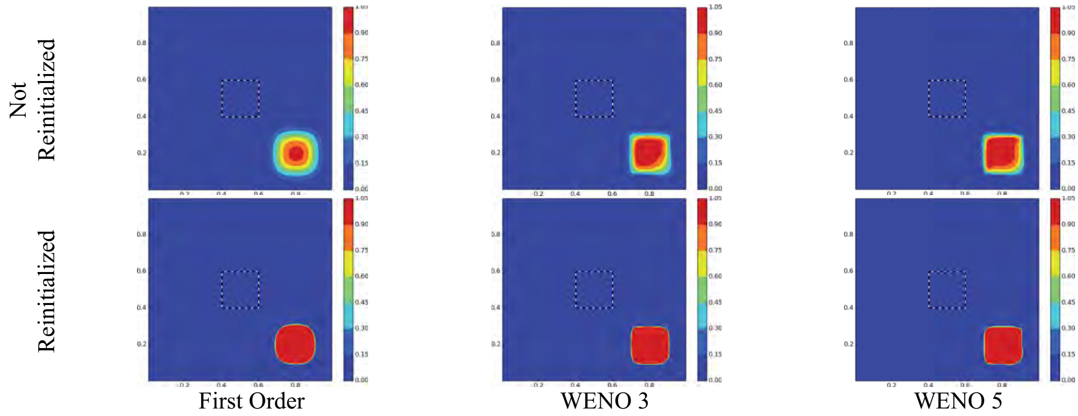


FIGURE 2. Translation of Square Volume

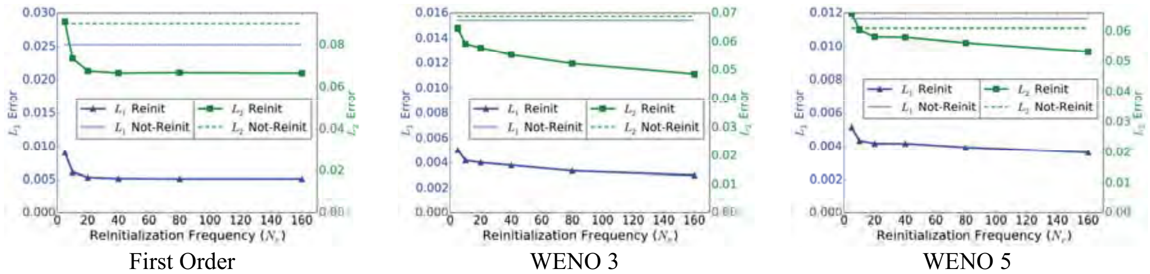


FIGURE 3. Error Plots - Translation of Square Volume

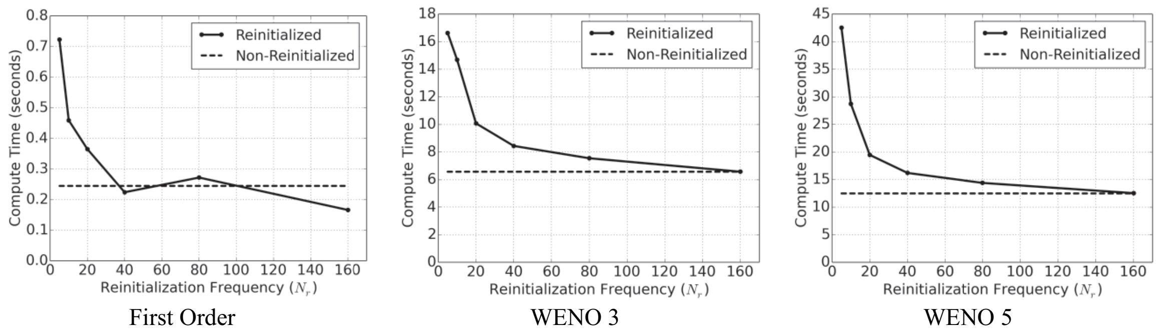


FIGURE 4. Computation Time - Translation of Square Volume

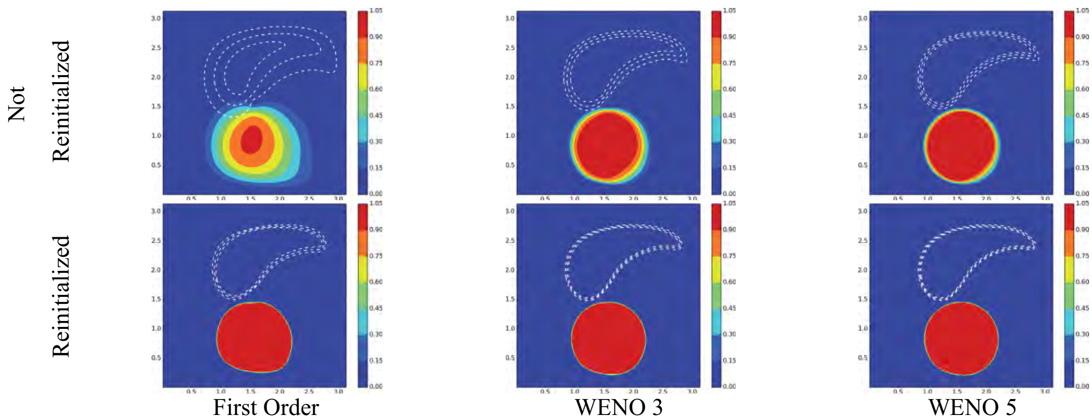
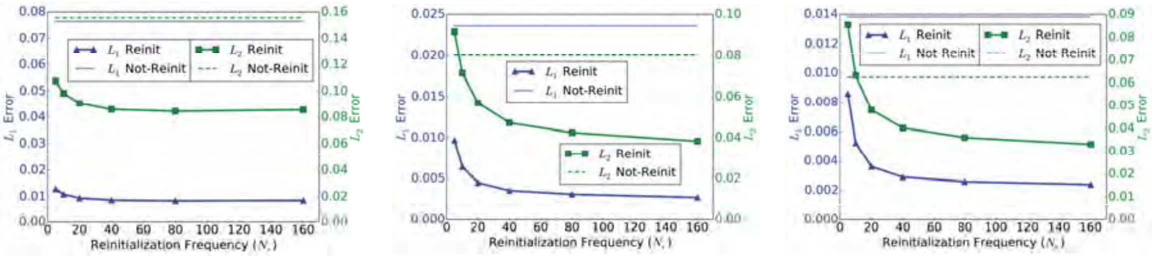
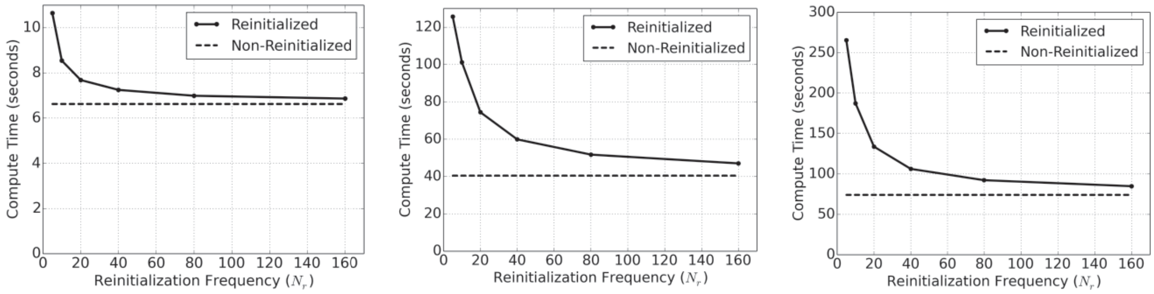


FIGURE 5. Circular Volume in Transient Shear Velocity Field



First Order WENO 3 WENO 5

FIGURE 6. Error Plots - Circular Volume in Transient Shear Velocity Field



First Order WENO 3 WENO 5

FIGURE 7. Computation Time - Circular Volume in Transient Shear Velocity Field

4. CONCLUSIONS

The commonly used techniques in the VOF method, using geometry based interface reconstruction, like PLIC, maintain very sharp fluid interfaces and also conserve mass. However, they are found to be complicated to extend to unstructured mesh, which is required for analysis of flow over complex geometries. The Riemann solver based methods, on the other hand, can be easily extended to unstructured mesh but suffer from the problem of diffused fluid interface. The reinitialization technique proposed here, overcomes the shortcomings of these two methods. This technique is based on the marching squares method which is commonly used in computer graphics and visualization. The new method is found to successfully conserve mass and restrict the diffusion error introduced by Riemann solver based methods. Also, this methodology can be easily extended to unstructured mesh. It is observed from solved test cases that the method maintains very sharp interfaces between the fluids. The high order solution reconstruction methods, do not diffuse the interfaces too much in few iterations, hence the reinitialization of the VOF function can be carried out less frequently to reduce computational cost.

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