A Novel Volume of Fluid Method for Interface Tracking

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Abstract. A finite volume based high resolution scheme, known as SDWLS, is proposed for the advection of volume fraction. The proposed scheme is then compared with other popular volume tracking methods. A few test problems are solved on Cartesian grid to demonstrate the efficacy of the proposed scheme. The new scheme does not use any special interface reconstruction for computation of interface. Consequently, the implementation on unstructured grid will be straightforward.

Keywords: Volume of Fluid, Interface Reconstruction, High Resolution Schemes, SDWLS

1 Introduction

Numerical computation of interface dynamics has been an interesting area from past few decades. It find many applications in engineering problems such as prediction of heat transfer through boiling [1], study of flows through weirs and spillways [2], study of water waves and ship dynamics [3] etc. Two common approaches to compute the dynamics of interface are, explicitly tracking the interface by moving the mesh along with the interface, referred as "interface tracking method", and employing an indicator function to capture the location of interface during evolution, referred as "interface capturing method". However, as compared to interface capturing type methods, application of interface tracking type methods on problems having complex / topological changes (such as, breaking up of air-water interface during spashing) are difficult.

The interface capturing type methods varies largely based on the choice of indicator function. For example, in case of volume of fluid method (VOF), the fractional volume on each cell is used as an indicator function [5]. However, for level set method, a signed distance function is being used [4]. In case of VOF, the interface is constructed through an interface reconstruction technique using the volume fraction and orientation of the fractional volume in each cell containing the interface (which is obtained from the neighbour cell volume fraction information). Furthermore, the volume fraction for the next time level is computed form the evolution of the constructed interface. Some of the popular techniques being used are SLIC (Simple Line Interface Calculation) [6], PLIC (Piecewise Linear

Interface Computation) [7] etc. The essential idea is to obtain the co-ordinates of the points through which the interface cut the cells. In contrast with level set method, VOF is inherently conservative and hence more suitable for simulating fluid flow problems. However, in case of arbitrary mesh, the interface reconstruction procedure will become extremely complicated.

A flux corrected transport approach for volume of fluid method (FCT-VOF) in [8] is another way to capture a non-diffusive interface without employing complicated interface reconstruction procedure. However, they are based on one dimensional formulation with extension towards multi-dimensions through sweeping on each dimensions by operator split. Hence, an unstructured extension of such methods are difficult. Moreover, the scheme results in formation of “jetsam” and “floatsam” [9].

The present study demonstrate a conservative finite volume based numerical scheme for accurate advection of volume fractions. Here, a high resolution weighted least square based method, called Solution Dependend Weighted Least Square method (SDWLS) [10] for advecting volume fraction is being used. Few test problems to demonstrate the efficacy of the proposed scheme are solved on a two dimensional Cartesian grid

2 Methodology

Let the fractional volume of fluid on each cell be $C \in [0, 1]$, such that $C = 1$ for cells which are completely filled with fluid and $C = 0$ for empty cells. Cells containing interface will have volume fraction between 0 and 1 as shown in Fig.1(d). The evolution of volume fraction is obtained by solving the following advection equation [5].

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = 0 \tag{1}$$

where, (u, v) denotes the fluid velocity in (x, y) Cartesian coordinate system. For a divergence-free velocity field, equation (1) can be re written as,

$$\frac{\partial C}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0 \tag{2}$$

where, $F = uC$ and $G = vC$

2.1 Finite Volume Discritization

Finite volume discritisation of Equation (2) using operator splitting [11] is given by,

$$C^{*}_{i,j} = C^{n}_{i,j} + \frac{\Delta t}{\Delta x}[F^{n}_{i-\frac{1}{2},j} - F^{n}_{i+\frac{1}{2},j}] \tag{3}$$

$$C^{n+1}_{i,j} = C^{*}_{i,j} + \frac{\Delta t}{\Delta y}[G^{*}_{i,j-\frac{1}{2}} - G^{*}_{i,j+\frac{1}{2}}] \tag{4}$$

where,

$$F^n_{i+\frac{1}{2},j} = \begin{cases} u_{i+\frac{1}{2},j} C^L_{i+\frac{1}{2},j} : u_{i+\frac{1}{2},j} \geq 0 \\ u_{i+\frac{1}{2},j} C^R_{i+\frac{1}{2},j} : u_{i+\frac{1}{2},j} < 0 \end{cases}$$

$u_{i+\frac{1}{2},j}$ is the normal velocity at cell interface $(i+\frac{1}{2},j)$ of cell (i,j). Similar expression can be derived for $G^n_{i,j+\frac{1}{2}}$ with normal velocity as $v_{i,j+\frac{1}{2}}$.

3 Flux Calculation

For calculating fluxes across each edge, two approaches for VOF method are followed in the present study. First approach is the interface reconstruction technique based on geometric interpretation and the other approach is solution reconstruction technique using high resolution scheme. Brief description of both the techniques are given in the following sections.

3.1 Interface Reconstruction based on geometric interpretation

In this approach, the volume fraction and its orientation on each cell is required for the calculation of fluxes. The two popular methods based on this approach are SLIC and PLIC. There are two varients in SLIC. First varient is one of the earlist known technique due to Noh and Woodward [6], where the rectangular blocks are either vertical or horizontal depending on the sweep direction. On the other hand, the second varient, due to Hirt and Nichols [5], uses both horizontal and vertical blocks. In the present study, the later version is used.

In PLIC [7], the interface is reconstructed using a piecewise linear approximation. Slope for defining the orientation of interface is calculated by Youngs method. Formulation of slope and shape of interface in a cell is explained in details by Rudman[8]. Fig.1 show a schematic representation of SLIC and PLIC. A detailed explanation of flux calculation from the reconstructed interface is given in [11].

3.2 Solution Reconstruction using High Resolution Scheme

Although a first order upwind scheme is non-oscillatory, the results are highly diffusive across abrupt changes. Rudman introduced FCT-VOF [8] using Zalesak's flux corrected approach [12]. However, the method introduces spurious oscillations in the solution, which create problem of flotsam / jetsam with distorted shape [9]. High resolution schemes are used to minimize the diffusion error, also have a better control on spurious oscillations, of the solution. A new second order high resolution scheme is introduced in the present study. A solution dependent weighted least square (SDWLS) method is used to reconstruct left and right side cell interface values in second order formulation of volume fraction [10]. A brief description of this method is given in this section.

Following Fig. 2, second order approximation of volume fraction at the cell interface $i+\frac{1}{2},j$ using left and right side values is given by

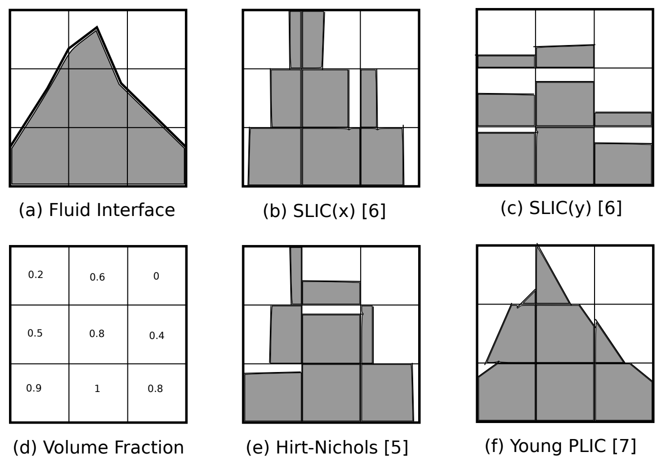


Fig. 1: Example of Interface Reconstruction Technique [8]

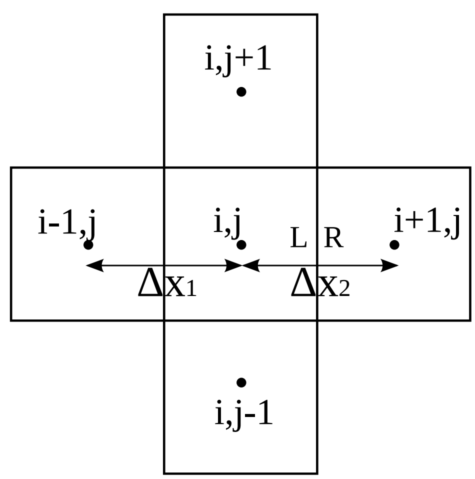


Fig. 2: 2D stencil for SDWLS formulation

$$C^L_{i+\frac{1}{2},j} = C_{i,j} + \left(\frac{\partial C}{\partial x}\right)_{i,j} (x_{i+\frac{1}{2},j} - x_{i,j}) \quad (5)$$

$$C^R_{i+\frac{1}{2},j} = C_{i+1,j} + \left(\frac{dC}{dx}\right)_{i+1,j} (x_{i+\frac{1}{2},j} - x_{i+1,j}) \quad (6)$$

where, $\left(\frac{\partial C}{\partial x}\right)_{i,j}$ is evaluated based on SDWLS gradient approximation [10] as

$$\left(\frac{\partial C}{\partial x}\right)_{i,j} = \frac{w_1 \Delta C_1 + w_2 \Delta C_2}{w_1 + w_2} \quad (7)$$

where, $\Delta C_1 = (C_{i-1,j} - C_{i,j})$, $\Delta C_2 = (C_{i+1,j} - C_{i,j})$, $w_1 = \frac{1}{\Delta C_1^2}$ and $w_2 = \frac{1}{\Delta C_2^2}$. Similarly, $\left(\frac{\partial C}{\partial x}\right)_{i+1,j}$ can also be evaluated.

Similar equations can be defined in y direction for $(i, j + \frac{1}{2})$ cell interface. For test cases, $u_{i+\frac{1}{2},j}$ and $v_{i,j+\frac{1}{2}}$ are computed from the specified distribution of u and v.

4 Results and Discussion

Three test cases are performed using SLIC and PLIC methods. The SDWLS results are then compared with both results.

4.1 Translation test

In this test, a square shaped and a circular shaped interface is advected under a uniform velocity field. Initial positions of square and circle is shown by dotted line in each plot. Velocity field is defined as $u = 1$, $v = 1$.

Plots for time $t = 0.4$ seconds is shown in Fig. 3. Square block performs better in SLIC, rather than PLIC and SDWLS. However, it fails to perform well in translation of circular block, since distortion at edges are quite visible in the plots. PLIC and SDWLS have maintained perfect shape in circular block. Both methods have produced the results with small diffusion at corners in square translation.

4.2 Rotational Test

Rotational test on a circular disk of radius 0.2 unit with slot of 0.1 unit width is performed. Velocity is defined in such a way that at mid point of edge of the boundary, its value is unity. Fig 4 shows the results after one complete rotation. In comparison with SLIC, both PLIC and SDWLS produce superior results. Distortion of edges are clearly visible in SLIC. SDWLS have slight diffusion at the corners. However shape is maintained in all the methods.

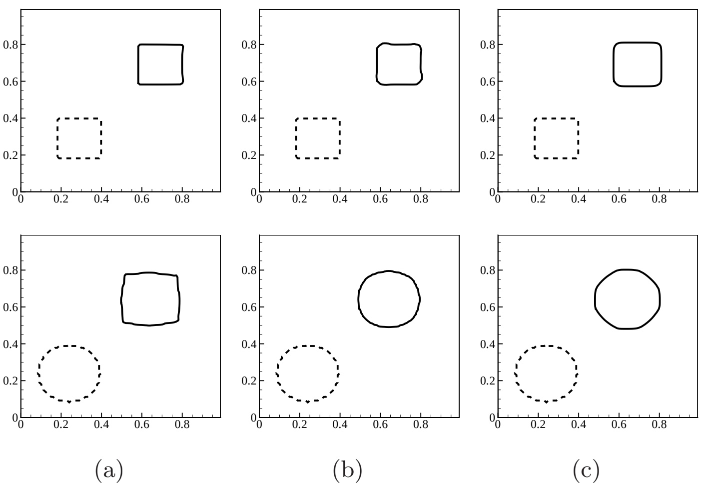


Fig. 3: Results for Translation of Square and Circular block after t =0.4 sec with grid 100x100 and CFL=0.25 for C=0.2 using (a) SLIC (b) PLIC (c) SDWLS.

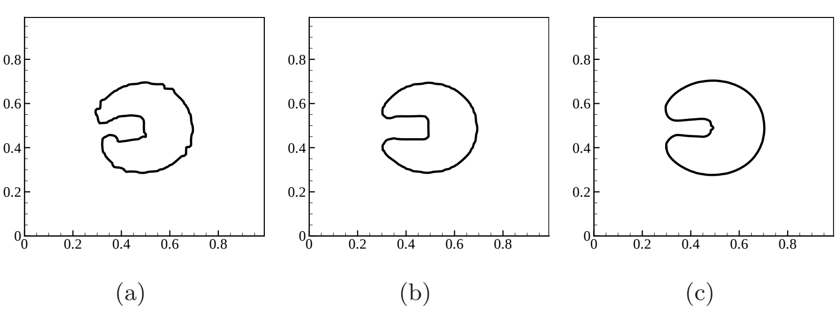
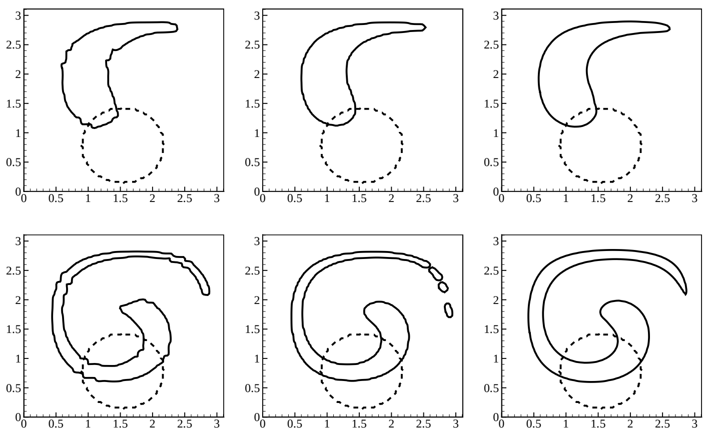


Fig. 4: Results for Rotation of Circular Disk after one complete rotation on 100x100 grid with CFL=0.25 for C=0.2 using (a) SLIC (b) PLIC (c) SDWLS.

4.3 Shear Test

Here, a circular interface is subjected to a shear deformation. A circle of $\pi/5$ radius is defined with center at location $(\pi/2, \pi/4)$ with computational domain as $\pi \times \pi$. The Velocity field is defined by

$$u = -\sin(x)\cos(y), \qquad v = \cos(x)\sin(y) \tag{8}$$



(a) (b) (c)

Fig. 5: Results for Shear of Circular Disk after $t = 5$ and 15 sec on 100x100 grid with CFL=0.25 for C=0.2 using (a) SLIC (b) PLIC (c) SDWLS.

Fig.5 shows the configuration after time $t = 5s$, and $t = 15s$. Initial position of circular block is shown by dotted lines in each plot. SDWLS produced a smooth interface while SLIC have distortions on the surface. Even though PLIC produces smooth interface, the formation of "floatsam" is clearly visible at the tail region.

5 Conclusion

Clearly from above results SDWLS maintains smooth interface with slight diffusion at the edges. SDWLS is easy to extend on unstructured grids, whereas both SLIC and PLIC is difficult to formulate. Since, SLIC depicts interface as rectangular blocks it has performed better in cases where edges are straight rather than circular, such as square translation. However, distortion of shape is quite visible for SLIC in all other test cases. PLIC is superior than all other methods in maintaining shape of interface. However, problem of jetsam is prominent in shear test at higher time levels. This problem of unphysical flows is eliminated in SDWLS. In all the above test cases SDWLS has manged to maintain shape without any distortion even on higher time levels with non uniform velocity fields.

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