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A Novel HLLC-Type Riemann Solver for Two-Phase Incompressible Flows

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Abstract

In simulation of two-fluid flows it is very important to capture the interface accurately. The interface in a two-fluid flow behaves like a contact observed in case of compressible fluid flows. Therefore, a contact preserving Riemann solver is likely to provide more accurate solution for two-fluid flows. A HLLC-type (Harten Lax van-Leer with contact) Riemann solver is proposed for incompressible Euler equations tightly coupled with the volume of fluid equation. The new solver is called HLLC-VOF Riemann solver. The efficacy of the new Riemann solver is tested by solving standard test problems involving two fluids. The results show an improved accuracy over a non-contact preserving Riemann solver.

Keywords: Contact Preserving Riemann Solver; Two-phase Flows; Volume of Fluid; Artificial Compressibility; HLLC-VOF

I. INTRODUCTION

There are various methods available for simulation of two fluid flows. The majority of these methods can be divided into two groups: the interface tracking methods and the volume tracking methods. In interface tracking methods the interface front is represented by a set of connected points. The volume tracking methods on the other hand, use a marker function to track the volume by using an additional advection equation. The merging and breaking of interfaces is handled naturally by these methods without a need of any additional treatment. The level-set (LS) method and the volume of fluid (VOF) method are two important methods in this category. Here, we have used the VOF method which uses the volume fraction as the marker function.

The steady state inviscid fluid flow problem is solved by using the artificial compressibility formulation. These equations are hyperbolic in nature, and therefore allow to use the advanced high order methods to solve the equations very accurately. The volume fraction advection equation is tightly coupled with the steady state fluid flow equations to obtain a two-fluid model. The unsteady equations are obtained by using a dual time stepping methodology.

The convective part of the equations is evaluated by using a novel Riemann solver which preserves the contact. The eigenvalue associated with the additional volume fraction equation has a value of material velocity. The contact preserving Riemann solver can therefore be assumed to provide better solution compared to non-contact preserving solvers. The results obtained by using the new Riemann solver are compared with HLL solver [1] and available results from literature, to access the efficacy of the new solver.

II. THE GOVERNING EQUATIONS

The governing equations, using artificial compressibility formulation and VOF, for a incompressible steady-state two-phase inviscid flow are given as,

$$
\begin{gathered}
\frac{\partial \mathbf{U}}{\partial \tau} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{S} \\
\mathbf{U} = \begin{bmatrix} p \\ \rho u \\ \rho v \\ C \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \beta u \\ \rho u^2 + p \\ \rho u v \\ uC \end{bmatrix}, \\
\mathbf{G} = \begin{bmatrix} \beta v \\ \rho u v \\ \rho v^2 + p \\ vC \end{bmatrix}, \quad \mathbf{S} = \begin{bmatrix} 0 \\ \rho g_x \\ \rho g_y \\ 0 \end{bmatrix};
\end{gathered} \tag{1}
$$

where β is a constant artificial compressibility paramater, u, v are the velocity components in x and y directions respectively and p is the pressure. The density is given by,

$$
\rho = \rho(C) = \rho_w C + \rho_a (1 - C). \tag{2}
$$

III. NUMERICAL FORMULATION

A. Finite volume discretization

The finite volume discretization formula for a M sided cell can be written as,

$$
\Omega_i \frac{\partial \overline{\mathbf{U}}}{\partial \tau} + \sum_{m=1}^{M} (\mathbf{F} n_x + \mathbf{G} n_y)_m \Gamma_m = \Omega_i \overline{\mathbf{S}}; \tag{3}
$$

where, n_x, n_y are the normals to each of the M sides, Ω is the area of the cell and Γ is the length of the cell sides.

B. Convective flux calculation

The solution reconstruction of the discrete data is done by using the fifth order weighted essentially non-oscillatory (WENO) method as described in [2].

The three-wave structure used for HLLC-VOF formulation is shown in figure 1. Based on the Rankine-Hugoniot jump conditions we can write the flux at the interface as,

$$\mathbf{F}^{HLLC-VOF} = \begin{cases} \mathbf{F}_L & 0 \le S_L \\ \mathbf{F}_{*L} = \mathbf{F}_L + S_L\left(\mathbf{U}_{*L} - \mathbf{U}_L\right) & S_L \le 0 \le S_* \\ \mathbf{F}_{*R} = \mathbf{F}_R + S_R\left(\mathbf{U}_{*R} - \mathbf{U}_R\right) & S_* \le 0 \le S_R \\ \mathbf{F}_R & S_R \le 0 \end{cases} \tag{4}$$

After carrying out mathematical manipulations of the Rankine-Hugoniot conditions across the three waves and the generalized Riemann invariants, we have obtained the following relations for the unknown intermediate states,

$$p_{*L} = p_{*R} = p_* = \frac{\beta(u_L - u_R) - S_L p_L + S_R p_R}{S_R - S_L}; \tag{5}$$

$$C_{*L} = \frac{C_L\left(S_L - u_L\right)}{S_L - S_*}; \quad v_{*L} = \frac{\left(S_L - u_L\right)\rho\left(C_L\right)v_L}{\left(S_L - S_*\right)\rho\left(C_{*L}\right)}; \tag{6}$$

$$C_{*R} = \frac{C_R\left(S_R - u_R\right)}{S_R - S_*}; \quad v_{*R} = \frac{\left(S_R - u_R\right)\rho\left(C_R\right)v_R}{\left(S_R - S_*\right)\rho\left(C_{*R}\right)}; \tag{7}$$

$$S_* = \frac{u_L\rho\left(C_L\right)\left(u_L - S_L\right) - u_R\rho\left(C_R\right)\left(u_R - S_R\right) - p_R + p_L}{\left(\rho_a - \rho_w\right)\left(C_R u_R - C_L u_L\right) + S_R\rho\left(C_R\right) - S_L\rho\left(C_L\right)}. \tag{8}$$

These equations can be used for calculation of U_{*L} and U_{*R}, which can then be used for calculation of required flux by substituting in (4).

C. Dual time stepping

The solution obtained from the artificial compressibility equations is only valid at the steady state. Since we are interested in simulation of unsteady problems, we have to use a dual time stepping strategy. The methodology used here is similar to the one used by Gaitonde in [3].

IV. RESULTS AND DISCUSSION

The volume of fluid equations coupled with the incompressible-Euler equations are solved by using the dual time stepping method. The two fluids considered are water ($\rho_w = 1000$) and air ($\rho_a = 1.125$). The acceleration due to gravity is taken as $(g_x, g_y) = (0, -9.8)$. Two of the tested problems are presented below and compared with results from literature.

A. Dam break problem

This is a classical two-phase test problem. In this problem, a column of water is placed initially towards the left-bottom corner in a rectangular domain as given in [4]. Under the influence of gravity the water column collapses and evolves into complicated free surface. The boundary condition for all the sides is set to slip walls. The experimental results for the location of the water fronts (left top and right) are available in literature [5]. The interface location is shown at 0.2, and 0.6 seconds in figure 2. The comparison of the results of the HLLC-VOF solver with HLL solver and the experimental data is shown in figure 3. In this plot h is the height of the interface at the left which is normalized by the initial height $b = 0.9$, l is the location of the front of the water surge which is normalized by the initial width of the water column $a = 0.45$. It can be seen that the HLLC-VOF solver fits the experimental data slightly better compared to the HLL solver.

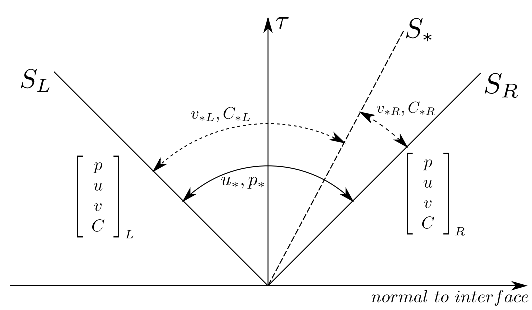

Figure 1: **HLLC-VOF wave structure**

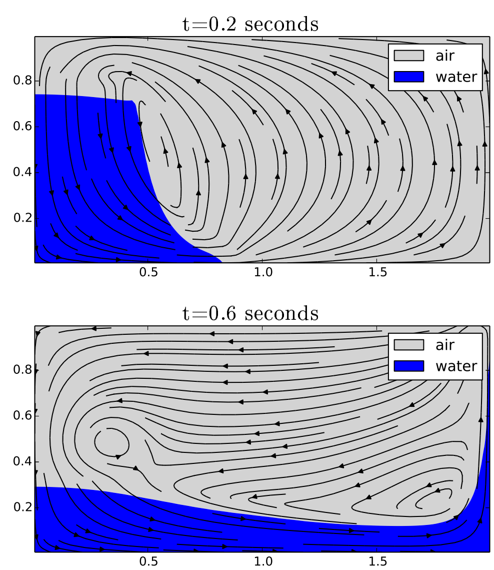

Figure 2: **Dam Break Problem**

B. Low amplitude sloshing problem

In this problem, the surface of water is set initially by a half-cosine function with an amplitude of 0.005 m and a mean height of 0.05 m as given in [4]. Under the influence of gravity the water surface starts sloshing inside the square box of dimensions 0.1 m $\times$ 0.1 m. The boundary condition for all the sides is set to slip walls. The first mode of oscillation can be estimated analytically [6] to be having a period of approximately 0.3739 seconds. The interface location is plotted at time 0.19 seconds in figure 4. The comparison of the theoretical first mode of oscillation with the y-location of interface at the left wall is shown in figure 5.

The frequency of oscillation is captured by both the solvers properly. The higher amplitude for odd peaks, compared to the analytical first mode, captured by HLLC-VOF solver is actually more accurate compared to HLL. This occurs due to the secondary mode of oscillations as described in [4].

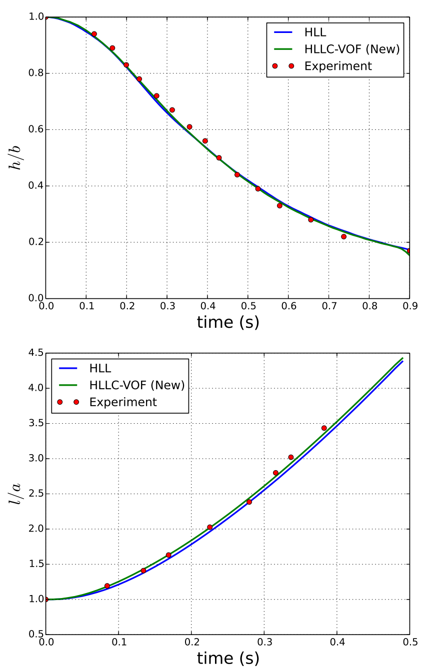

Figure 3: **Dam break - height and surge position**

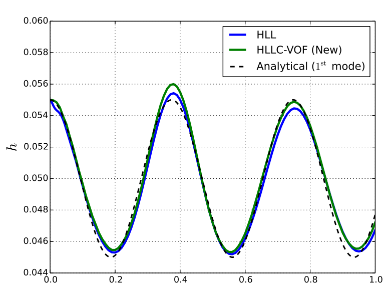

Figure 5: **Low amplitude sloshing - first mode**

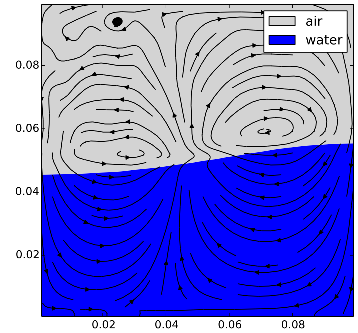

Figure 4: **Low amplitude sloshing (t=0.19 seconds)**

V. CONCLUSION

A new contact preserving HLLC-type Riemann solver is developed for incompressible two-phase Euler equations. The new solver is tested by solving standard test problems. It is observed that the HLLC-VOF Riemann solver provides accurate solutions compared to a non-contact preserving Riemann solver.

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