1.9) A small turbo-jet engine uses 50 kg/s of air, and the air/fuel ratio is 90 : 1. The jet efflux velocity is 600 m/s. When the afterburner is used, the overall air/fuel ratio decreases to 50 : 1 and the jet efflux velocity increases to 730 m/s. Find the static thrust with and without the afterburner. The pressure on the engine discharge plane can be assumed to be equal to the ambient pressure in both cases.

Solution:

Given: $p_1 = p_2$

case 1 - without afterburner: $\dot{m}_1 = 50\,\text{kg/s}$, $\dot{m}_1 : \dot{m}_{\text{fuel}} = 90 : 1$, $V_2 = 600\,\text{m/s}$.

case 2 - with afterburner: $\dot{m}_1 = 50\,\text{kg/s}$, $\dot{m}_1 : \dot{m}_{\text{fuel}} = 50 : 1$, $V_2 = 730\,\text{m/s}$.

To calculate: static thrust for case 1 and case 2.

The schematic diagram of the problem description is shown in Fig. 1.

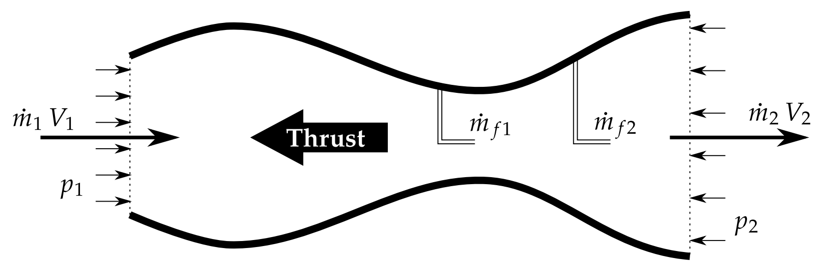

Fig. 1: Schematic diagram for problem description

Case1: without afterburner

Using the conservation of mass,

$$\dot{m}_2 = \dot{m}_1 + \dot{m}_{f1} = \dot{m}_1 + \dot{m}_1/90 = 50 + 50/90 = 50.556\,\text{kg/s}.$$

Assuming $V_1 \sim 0$ (since engine is stationary on ground),

The conservation of momentum gives,

$$\begin{aligned}
\text{Thrust} &= \text{rate of momentum exiting} - \text{rate of momentum entering} \\
&\quad + \text{pressure force at exit} - \text{pressure force at inlet} \\
\text{Thrust} &= \dot{m}_2 V_2 - \dot{m}_1 V_1 + (p_2 - p_1)\, A_{\text{exit}} \\
\text{Thrust} &= 50.555556 \times 600 \\
\text{Thrust} &= 30333.3336\,\text{N}
\end{aligned}$$

$$\boxed{\text{Static thrust} = 30333.3336\,\text{N}}\,.$$

Case2: with afterburner

Using the conservation of mass,

$$\dot{m}_2 = \dot{m}_1 + \left(\dot{m}_{f1} + \dot{m}_{f2}\right) = \dot{m}_1 + \dot{m}_1/50 = 50 + 50/50 = 51\,\text{kg/s}.$$

Assuming $V_1 \sim 0$ (since engine is stationary on ground).

The conservation of momentum gives,

$$\begin{aligned}
\text{Thrust} &= \text{rate of momentum exiting} - \text{rate of momentum entering} \\
&\quad + \text{pressure force at exit} - \text{pressure force at inlet} \\
\text{Thrust} &= \dot{m}_2 V_2 - \dot{m}_1 V_1 + (p_2 - p_1)\, A_{\text{exit}} \\
\text{Thrust} &= 51 \times 730 \\
\text{Thrust} &= 37230\,\text{N}
\end{aligned}$$

$$\boxed{\text{Static thrust} = 37230\,\text{N}}\,.$$