1.1) An air stream enters a variable area channel at a velocity of 30 m/s with a pressure of 120 kPa and a temperature of 10° C. At a certain point in the channel, the velocity is found to be 250 m/s. Using Bernoulli's equation (i.e. $p + \rho V^2/2 = \text{constant}$), which assumes incompressible flow, find the pressure at this point. In this calculation, use the density evaluated at the inlet conditions. If the temperature of the air is assumed to remain constant, evaluate the air density at the point in the flow where the velocity is 250 m/s. Compare this density with the density at the inlet to the channel. Based on this comparison, do you think that the use of Bernoulli's equation is justified?

Solution:

The problem description is schematically shown in Fig. 1.

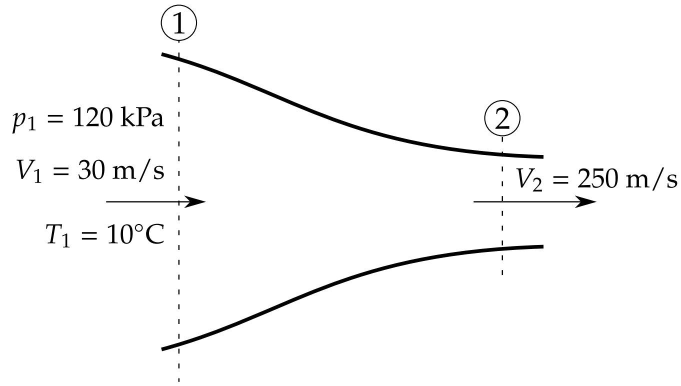

Fig. 1: Schematic diagram for describing the problem

Since the flow is assumed to be incompressible, ρ is constant and can be calculated at station (1) using ideal gas equation,

$$\rho = \rho_1 = \frac{p_1}{R\,T_1} = \frac{120 \times 10^3}{287 \times (10 + 273)} = 1.47745\,\text{kg/m}^3$$

Using the Bernoulli's equation

$$p_1 + \rho\,\frac{V_1^2}{2} = p_2 + \rho\,\frac{V_2^2}{2}$$

which can be used to solve for pressure at station (2)

$$p_2 = p_1 + \frac{\rho}{2}\left(V_1^2 - V_2^2\right) \tag{1}$$

Substituting $\rho = 1.47745\,\text{kg/m}^3$ in (1), the pressure at station (2) can be calculated as,

$$p_2 = p_1 + \frac{\rho}{2}\left(V_1^2 - V_2^2\right)$$

$$p_2 = 120 \times 10^3 + \frac{1.47745}{2}\left(30^2 - 250^2\right) \quad = \quad 74494.54\,\text{Pa}$$

$$\boxed{p_2|_{\text{incomp.}} = 74.49454\,\text{kPa}}$$

If the temperature of the air is assumed to remain constant, $T_2 = T_1 = 283\,\text{K}$, then we can calculate the density at station (2) to be,

$$\rho = \frac{p_2}{R\,T_2} = \frac{74494.54}{287 \times 283}$$

$$\boxed{\rho|_{T=\text{constant}} = 0.917183\,\text{kg/m}^3}$$

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The percentage difference in calculated density, at station (1) and assuming constant temperature is,

$$\begin{aligned}\text{\% difference in density calculation} &= \frac{\rho_1 - \rho|_{T=\text{constant}}}{\rho|_{\text{Bernoulli}}} \times 100 \\ &= \frac{1.47745 - 0.917183}{1.47745} \times 100 \\ &= 37.92\%\end{aligned}$$

Based on this comparison, it is clear that the use of Bernoulli's equation is **not** justified, as in an incompressible flow (with no heat transfer) the temperature variations are negligible. Therefore, the density calculated with this assumption must closely match with the result of Bernoulli's equation if the flow is indeed incompressible.

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